Skm's Jashbhai Maganbhai Patel College of Commerce

Department : B.Sc.I.T.

Programme: F. Y. B. Com. (SEM-II)

Course: Mathematical Statistical Techniques-II

Interest & Annuity

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INTEREST AND ANNUITY

1. Simple Interest
2. Compound Interest
3. Annuity
4. Equated Monthly Instalment
5. Reducing Balance Method
6. Flat Rate Method
7. Stated Annual Rate
8. Depreciation
9. Effective Annual Rate

Three Payment Methods

Loan payments can be structured in one of 3 ways

Discount Loan

Principal and interest is paid in lump sum at the end

Interest-only loan

Periodic interest-only payments, principal due at end.

Amortized loan

Equal periodic payments of principal and interest

Simple Interest

$$S.I. = \frac{P \times n \times r}{100}$$

OR

S.I.= Pni

where i=r/100

A= P+S.I.

Where

P: Principal- The sum borrowed by a person

n: Period-The time spent for which money is lent

r: Rate of interest-This is the interest to be paid on the amount of Rs. 100/- **per annum** (p.a.)

I: Interest- The amount paid by a borrower to the lender for the use of money borrowed for certain period of time.

A: **Total Amount / Accumulated amount**- the sum of the principal and interest

Accumulated amount: A=P+SI

$$S.I. = \frac{P \times n \times r}{100}$$

Eg. : P=10000 r=8%pa n=1yr

100+8

10000+800 = 10800

Compound Interest

If P is principal , r is the rate of interest p.a. then amount at the end of “n” year called “Compound amount” and is given by

$$A = P\left(1 + \frac{r}{100}\right)^{n}$$

The Compound interest is given by **C.I.= A - P**

The interest may be compounded annually, semi-annually (half-yearly), quarterly, monthly

$$A = P\left(1 + \frac{r}{q \times 100}\right)^{nq}$$

q: the number of time interest is compounded

q=1 if interest compounded annually

$$A = P\left(1 + \frac{r}{100}\right)^{n}$$

q=2 if interest compounded semi-annually (half-yearly)

$$A = P\left(1 + \frac{r}{200}\right)^{2n}$$

q= 4 if interest compounded quarterly

$$A = P\left(1 + \frac{r}{400}\right)^{4n}$$

q=12 if interest compounded monthly

$$A = P\left(1 + \frac{r}{1200}\right)^{12n}$$

If Ms. Sonam has borrowed 8000 for 3 year at the rate of 6%p.a. then how much Simple interest she will pay?

P= 8000

N= 3

R= 6 % p.a. (OR 6 p.p.a)

$$S.I. = \frac{P * N * R}{100}$$

$$S.I. = \frac{8000 * 3 * 6}{100}$$

$$\mathbf{S.I. = 1440}$$

Mrs. Prabhu lent a total of Rs. 48000/- to Mr. Diwakar at 9.5% for 5 years and to Mr. Ratnakr at 9% for 7 years. If she receives a total interest of Rs. 25590, find the amount she lent to both.

Mrs. Prabhu : 48000 ; s.i= 25590

Diwakar : $P=P_1$; $n_1=5y$; $r_1=9.5\%$p.a. : $s_1 = \frac{P_1 * n_1 * r_1}{100}$

Ratnaakar : $P=P_2$; $n_2=7y$; $r_2=9\%$p.a. : $s_2 = \frac{P_2 * n_2 * r_2}{100}$

$$S.I. = \frac{P * N * R}{100}$$

$S= s_1+s_2$

$$S = \frac{P_1 * n_1 * r_1}{100} + \frac{P_2 * n_2 * r_2}{100}$$

$$S = \frac{P_1 * n_1 * r_1}{100} + \frac{P_2 * n_2 * r_2}{100}$$

$$S = \frac{P_1 * n_1 * r_1 + P_2 * n_2 * r_2}{100}$$

$$25590 = \frac{P_1 * 5 * 9.5 + P_2 * 7 * 9}{100}$$

$$25590 * 100 = P_1 * 5 * 9.5 + P_2 * 7 * 9$$

$$25590 * 100 = P_1 * 47.5 + P_2 * 63$$

$$2559000 = 47.5P_1 + 63P_2$$

$$2559000 = 47.5P_1 + 63(48000 - P_1)$$

Mrs. Prabhu lent a total of Rs. 48000

Therefore P= P_1+P_2

48000= P_1+P_2

P_2 = 48000-P_1

$$2559000 = 47.5P_1 + 63(48000 - P_1)$$

$$2559000 = 47.5P_1 + 63 * 48000 - 63 * P_1$$

$$2559000 = 47.5P_1 + 3024000 - 63\ P_1$$

$$2559000 - 3024000 = 47.5P_1 - 63\ P_1$$

$$-465000 = -15.5P_1$$

$$P_1 = \frac{-465000}{-15.5}$$

$$\boldsymbol{P_1 = 30000}$$

therefore Principal lent to Diwakar = $\boldsymbol{P_1 = 30000}$

Principal lent to Ratnakar = $\boldsymbol{P_2 = 48000 - P_1}$

Principal lent to Ratnakar = $\boldsymbol{P_2 = 48000 - 30000}$

Principal lent to Ratnakar = $\boldsymbol{P_2 = 18000}$

Miss. Kansar lent Rs. 2560/- to Mr. Abhijeet and Rs. 3650/- to Mr. Ashwin at 6% rate of interest. After how many years should she receive a total interest of Rs. 3726/-.

Ms. Kansar ---- $s=s_1+s_2=3726$

Mr. Abhijeet : $p_1=2560$; $r_1= 6$p.p.a ; $n_1=n$;

Mr. Ashwin: $p_2= 3650$; $r_2= 6$.p.p.a ; $n_2=n$

Abhijeet : P=2650 ; n_1 = n ; r_1=6%p.a. : $s_1 = \frac{P_1 * n_1 * r_1}{100}$

Ashwinr : P=3650 ; n_2= n ; r_2=6%p.a. : $s_2 = \frac{P_2 * n_2 * r_2}{100}$

$$\text{S. I.} = \frac{\text{P} * \text{N} * \text{R}}{100}$$

S= s_1+s_2

$$S = \frac{P_1 * n_1 * r_1}{100} + \frac{P_2 * n_2 * r_2}{100}$$

$$3726 = \frac{2650 * n * 6 + 3650 * n * 6}{100}$$

$$3726 * 100 = 2650 * n * 6 + 3650 * n * 6$$

$$372600 = 15360\ n + 21900\ n$$

$$372600 = 37260n$$

$$n = 10$$

Find the amount received when a sum of Rs. 5000 is invested at 10% p.a. for 3 years , if the interest is compounded

i) annually ii) half yearly iii) quarterly

$$A = P\left(1 + \frac{r}{q \times 100}\right)^{nq}$$

q: the number of time interest is compounded.

Annually: 1. q=1 if interest compounded annually

$$A = P\left(1 + \frac{r}{100}\right)^{n}$$

$$A = 5000\left(1 + \frac{10}{100}\right)^3$$

$$A = 5000\,(1 + 0.1)^3$$

$$A = 5000\,(1.1)^3$$

$$A = 5000 * 1.331$$

$$\boxed{A = 6655}$$

Half yearly: q=2 if interest compounded half yearly $A = P(1 + \frac{r}{200})^{2n}$

$$A = 5000\left(1 + \frac{10}{200}\right)^{2*3}$$

$$A = 5000\,(1 + 0.05)^6$$

$$A = 5000\,(1.05)^6$$

$$A = 5000 * 1.3400$$

$$\boxed{A = 6700}$$

Half yearly: q=4 if interest compounded quarterly

$$A = P\left(1 + \frac{r}{400}\right)^{4n}$$

$$A = 5000\left(1 + \frac{10}{400}\right)^{4*3}$$

$$A = 5000\,(1 + 0.025)^{12}$$

$$A = 5000\,(1.025)^{12}$$

$$A = 5000 * 1.3448$$

$$\boxed{A = 6724}$$

Half yearly: q=4 if interest compounded quarterly $A = P(1 + \frac{r}{1200})^{12n}$

$$A = 5000\left(1 + \frac{10}{400}\right)^{12*3}$$

$$A = 5000\,(1 + 0.008)^{36}$$

$$A = 5000\,(1.008)^{36}$$

$$A = 5000 * 1.3322$$

$$\boxed{A = 6661}$$

A bank offers fixed deposit for 5 years under the following schemes.

i) AT 15% , if the interest to be calculated half-yearly

ii) At 12%, if the interest is to be calculated quarterly

Stat which scheme is more beneficial to the public.

Mandar invested a certain amount for 3 years at 4% p.a. and got a simple interest of Rs. 1600.

He kept then kept aside the interest and once again invested the same amount at the compound interest of 10% p.a. for another 4 years.

If the compound interest is to be calculated annually,

find the final amount he receives at the end of the second deal. Also calculate his compound interest.

ANNUITY

•A strategy for saving a little bit of money in the present and having a big payoff in the future is called an **annuity.**

•An annuity is an account in which equal regular payments are made.

•Series of payments at fixed intervals, guaranteed for a fixed number of years or the lifetime of one or more individuals.

•An **annuity** is a financial contract written by an insurance company that provides for a series of guaranteed payments, either for a specific period of time or for the lifetime of one or more individuals.

Examples: premium of insurance policies, loan instalments, monthly recurring deposits etc.

There are two types of annuity

1. The ordinary annuity: The annuity which is paid at the end of each period is called as "Immediate annuity OR Ordinary Annuity".

2. Annuity Due: A cash flow stream such as rent, lease, and insurance payments, which involves equal periodic cash flows that begin right away or at the beginning of each time interval is known as an *annuity due.*

Formula Adjustment

PV annuity due = PV ordinary annuity x (1+r)

FV annuity due = FV ordinary annuity x (1+r)

PV annuity due > PV ordinary annuity

FV annuity due > FV ordinary annuity

There are two types of annuity

1.<u>The ordinary annuity:</u> The annuity which is paid at the end of each period is called as “Immediate annuity OR Ordinary Annuity”.

Eg. : How much money will you accumulate by the end of year 10 if you deposit Rs. 3,000/- each for the next ten years in a savings account that earns 5% per year?

$$A = P\left(1 + \frac{r}{100}\right)^{n}$$

YEAR	ANNUITY	RETURN	
1	3000	3000*(1+5/100)	3150
2	3000	$3000*(1+5/100)^2$	3307.5
3	3000	$3000*(1+5/100)^3$	3472.88
4	3000	$3000*(1+5/100)^4$	3646.52
5	3000	$3000*(1+5/100)^5$	3828.84

$$\mathbf{A = \frac{P}{i}\,[\,(1+i)^{n} - 1]}$$

FUTURE VALUE **/ ACCUMULATED VALUE**

Manoj opened a recurring deposite in a bank for 7 years with payment of Rs. 6000 paid at the end of each year. Find the money obtained at the end of period with 8% p.a.?

n=7 , P=6000, r=8%p.a. A= ?

I =r/100= 8/100 = 0.08

$$\mathbf{A = \frac{P}{i}\,[\,(1+i)^n - 1]}$$

A = (6000/0.08) * [(1+0.08)^7 -1]

A =(75000) * [1.7138 -1]

A = 75000 * 0.7138

A = 53536.8

6000/0.08 = 75000

(1+0.08) = 1.08

(1+0.08)^7 = 1.08 *1.08 *1.08*1.08*1.08*1.08*1.08 = 1.7138

What is the accumulated value after 3 years of an immediate annuity of Rs. 9000 p.a. , the rate of interest being 9% p.a.?

P=9000, n = 3 , r= 9%p.a.

$$\mathbf{A = \frac{P}{i}\,[\,(1+i)^n - 1]}$$

i= r/100 = 9/100 = 0.09

A = (9000/0.09) * [$(1+0.09)^3$ -1]

A =(100000)*[1.2950-1]

A=100000*0.2950

A= 29500

Ram deposited 5000 at the end of each year, for 2 years in a company and received 13,500 as the accumulated value , Find rate of compound interest?

P=5000, n=1, A= 13500, r=?

$$\mathbf{A = \frac{P}{i}\,[\,(1+i)^n - 1]}$$

$13500 = (5000/i)* [(1+i)^2-1]$

$13500/5000 = (1/i) [1+2i + i^2 -1]$

$2.7 = (1/i) *(2i+i^2)$

$2.7 = (1/i) * i (2+i)$

$2.7 = 2+i$

$2.7-2=i$ $\quad$ $i=0.7$

Let a loan of Rs 80000 is to be repaid in 2 years at 12% p.a. on reducing balance method calculate EMI and construct Amortization table

$$\mathbf{P.V.} = \frac{\mathbf{C}}{\mathbf{i}}\left[\mathbf{1} - \frac{\mathbf{1}}{(\mathbf{1+i})^{\mathbf{n}}}\right]$$

PV= 80000

n=2 years= 2*12=24months

r=12p.p.a.

i=r/12*100 = 12/1200 = 0.01

$$\mathbf{P.V.} = \frac{\mathbf{C}}{\mathbf{i}}\left[\mathbf{1} - \frac{\mathbf{1}}{(\mathbf{1+i})^{\mathbf{n}}}\right]$$

$$\mathbf{80000} = \frac{\mathbf{C}}{\mathbf{0.01}}\left[\mathbf{1} - \frac{\mathbf{1}}{(\mathbf{1+0.01})^{\mathbf{24}}}\right] \qquad \mathbf{80000} = \frac{\mathbf{C}}{\mathbf{0.01}}\left[\mathbf{1} - \frac{\mathbf{1}}{(\mathbf{1.01})^{\mathbf{24}}}\right]$$

$$80000 = \frac{C}{0.01}\,[\,0.2124\,]$$

$$80000 = c * \frac{0.2124}{0.01}$$

$$80000 = c * 21.24$$

$$c = \frac{80000}{21.24}$$

C= 3764.7058

EMI= 3764.70

1+0.01=1.01

1.01^24= 1.2697

1/(1.01^24)= 0.7876

1-1/(1.01^24)= 0.2124

Amortization Table

BREAK-UP OF EMI

Month	Principal	EMI	Interest part (i=0.01)	Principal Part	Outstanding principal
1	80000	3766.48	80000*0.01 =800	3766.48-800= 2966.48	80000-2966.48= 77033.52
2	77033.52	3766.48	77033.52*0.01 =770.33	3766.48-770.33 =2996.14	77033.52-2996.14= 74037.36
3	74037.36	3766.48	740.3736	3026.1064	71011.2536
4	67954.882	3766.48	679.548825	3086.931175	64867.95133

5	67954.88	3766.48	679.548825	3086.931175	64867.95133
6	64867.95	3766.48	648.6795133	3117.800487	61750.15084
7	61750.15	3766.48	617.5015084	3148.978492	58601.17235
8	58601.17	3766.48	586.0117235	3180.468277	55420.70407
9	55420.7	3766.48	554.2070407	3212.272959	52208.43111
10	52208.43	3766.48	522.0843111	3244.395689	48964.03542
11	48964.04	3766.48	489.6403542	3276.839646	45687.19578
12	45687.2	3766.48	456.8719578	3309.608042	42377.58773
13	42377.59	3766.48	423.7758773	3342.704123	39034.88361
14	39034.88	3766.48	390.3488361	3376.131164	35658.75245
15	35658.75	3766.48	356.5875245	3409.892476	32248.85997
16	32248.86	3766.48	322.4885997	3443.9914	28804.86857
17	28804.87	3766.48	288.0486857	3478.431314	25326.43726
18	25326.44	3766.48	253.2643726	3513.215627	21813.22163
19	21813.22	3766.48	218.1322163	3548.347784	18264.87385
20	18264.87	3766.48	182.6487385	3583.831262	14681.04258
21	14681.04	3766.48	146.8104258	3619.669574	11061.37301
22	11061.37	3766.48	110.6137301	3655.86627	7405.506741
23	7405.507	3766.48	74.05506741	3692.424933	3713.081808
24	3713.082	3766.48	37.13081808	3729.349182	-16.26737399

EMI by Flat Rate of interest

$$\text{EMI} = \frac{P\left(1 + \frac{nr}{100}\right)}{12n}$$

$$\text{EMI} = \frac{P\,(1 + ni)}{12n}$$

Using Flat rate of interest calculate EMI on Rs. 10000 at 6%p.a. for 3 years

P=10000
N= 3YEARS
R= 6 P.P.A.

$$\text{EMI} = \frac{P\,(1 + ni)}{12n}$$

$$\text{EMI} = \frac{10000\,(1 + 3 * 0.06))}{12 * 3}$$

$$\text{EMI} = \frac{11800}{36} = 327.76$$

3 x 0.06 = 0. 18
1 + 0.18 = 1.18
10000 x 1.18 = 11800
12 x 3 = 36
11800 / 36

Depreciation:

Depreciation value = $P(1-i)^n$

Q. 1) A machinery of 500000 was purchased by a manufacturer. Find out its price for 5 years by considering 10% depreciation for each year.

original price = P =500000

i=r/100 = 10/100 = 0.1

year	Depreciation value	loss	value after depreciation
1	500000	500000*10% = 50000	500000-50000 =450000
2	450000	450000*10% = 45000	450000-45000= 405000
3	405000	405000*10% = 40500	405000-40500= 364500
4	364500	364500*10% = 36450	364500-36450 = 328050
5	328050	328050*10% = 32805	328050-32805= 295245

Depreciation:

Depreciation value = $P(1-i)^n$

Q. 1) A machinery of 100000 was purchased by a manufacturer. Find out its price after 3 years by considering 20% depreciation for each year.

original price = P =100000

i=r/100 = 20/100 = 0.2

year	Value before Depreciation	loss	value after depreciation
1	100000	100000*20% = 20000	100000-20000 =80000
2	80000	80000*20% = 16000	80000-16000= 64000
3	64000	64000*20% = 12800	64000-12800= 51200

Q.2) A second hand motor cycle is priced at 55016 after 2 years with 8% depreciation p.a. find its original price.

Depreciated value = 55016, n =2, r=8%p.a.

Depreciation value = Original Value$(1-i)^n$

Depreciation value = $P(1-i)^n$

P = Depreciation $/(1-i)^n$

P = 550106 / $(1-0.08)^2$

P = 65000

A company sets aside a 15000 annually to enable it to pay off a debenture issues of 180000 at the end of 10 years. Assuming that the sum accumulates at 6% p.a find the surplus after paying of the debenture stock?

$$A = \frac{P}{i}\left[(1+i)^n - 1\right]$$

P = 15000 , n= 10 years , r = 6%p.a. so I = 0.06

$$A = \frac{15000}{0.06}\left[(1+0.06)^{10} - 1\right]$$

$$A = 25000\left[(1.06)^{10} - 1\right]$$

$$A = 25000\left[1.7908 - 1\right]$$

$$A = 25000\left[0.7908\right]$$

$$A = 197700]$$

Effective rate of interest

The annual rate of compounding is called **Stated (Nominal) Rate of interest** and

The **Effective Rate of interest** is the rate of interest that investor can earn (or pay) in a year after taking into consideration compounding.

It is denoted by Re and is calculated as

$$R_e = \left(1 + \frac{i}{m}\right)^m - 1$$

Ms. Bhatia deposits a certain amount at the end of every year for 5 years in a bank. The rate of interest is 10% p.a. compounded half yearly, find the effective rate of interest

r= 10% p.a. = 0.1

m = 2 : periods in a year

$$R_e = (1 + i/m)^m - 1$$

$$R_e = (1 + 0.1/2)^2 - 1$$

$$R_e = (1 + 0.05)^2 - 1$$

$$R_e = (1.05)^2 - 1$$

$$R_e = 1.025 - 1$$

$$R_e = 0.025$$

A company sets aside a 15000 annually to enable it to pay off a debenture issues of 180000 at the end of 10 years. Assuming that the sum accumulates at 6% p.a find the surplus after paying of the debenture stock?

$$A = \frac{P}{i}\,[\,(1+i)^n - 1\,]$$

P = 15000 , n= 10 years , r = 6%p.a. so I = 0.06

$$A = \frac{15000}{0.06}\,[\,(1+0.06)^{10} - 1\,]$$

$$A = 250000\,[\,(1.06)^{10} - 1\,]$$

$$A = 250000\,[\,1.7908 - 1\,]$$

$$A = 250000\,[\,0.7908\,]$$

$$A = 1997700]$$

The surplus amount after paying the debenture stock = 197700-180000 = 17700

Mehta Housing societies has 8 members and collect 2500 as a maintenances charge from every member per year . The rate of compound interest is 8% p.a. if after 4 years the society needs to do a work worth 100000 are the annual charges enough to bare the cost?

$$A = \frac{P}{i}\,[\,(1+i)^n - 1\,]$$

P= ?, n = 4 , r = 8 % p.a. , i= 0.08

$$100000 = \frac{P}{0.08}\,[\,(1+0.08)^4 - 1\,]$$

$$100000 = \frac{P}{0.08}\,[\,(1.08)^4 - 1\,]$$

$$100000 = \frac{P}{0.08}\,[\,1.3064 - 1\,]$$

$$100000 = \frac{P}{0.08}\,[\,0.3064\,]$$

$$100000 = P\,(3.83)]$$

$$P = \frac{100000}{3.83}$$

$$\mathbf{P = 26109.6695}$$

Annual payment of all the 8 members should be 22197 = 22197/8= **2774.625**

This payment is less than Rs. 2500 which the society has decided to take presently.

Thus, the society should increase the annual sinking fund.